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Titles and Abstracts

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Recent Results on Omega Invariant and Realizability

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In this talk, We will discuss some recent results on omega invariant and realizability.

On the eigenvalues of the distance Laplacian matrix of graphs

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Let $G(V,E)$ be a simple graph of order n, size m and having the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. The transmission $Tr_G(v)$ of a vertex v is defined to be the sum of the distances from v to all other vertices in G, i.e., $Tr_G(v) = \sum_{u \in V(G)} d_{uv}$. Let $Tr(G) = diag(Tr_1, Tr_2, \ldots, Tr_n)$ be the diagonal matrix of vertex transmissions of G. The distance matrix of G is denoted by $D(G)$ and is defined as $D(G) = (d_{uv})u, v \in V(G)$. The matrix $DL(G) = Tr(G) - D(G)$ is called the distance Laplacian matrix of G. The eigenvalues of $DL(G)$ are called the distance Laplacian eigenvalues of G. Its eigenvalues can be ordered as $\delta_n^L(G) \geq \delta_{n-1}^L(G) \leq \ldots \leq \delta_1^L(G)$. The largest eigenvalue $\delta_1^L(G)$ is called the distance Laplacian spectral radius of G. We discuss the distribution of the distance Laplacian eigenvalues of graphs, the bounds for the distance spectral radius and the multiplicity of some distance Laplacian eigenvalues.

Graph Coloring and Its Variations

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By a proper vertex coloring of graph $G = (V, E)$, we mean to assign colors to the vertices of G such that adjacent vertices must get different colors. Minimum number of colors required to get a proper vertex coloring of G is known as chromatic number of the graph G and denoted by $\chi(G)$. A bijection $f : E \to \{1, 2, \ldots, |E|\}$ is called a local antimagic labeling if for any two adjacent vertices u and v, $w(u) \neq w(v)$, where $w(u) = \sum_{e \in E(u)} f(e)$, and $E(u)$ is the set of edges incident to u. It is clear that this local antimagic labeling induces a proper vertex coloring of G where the vertex v is assigned the color $w(v)$.

The local antimagic chromatic number $\chi la(G)$ is the minimum number of colors used over all colorings of G. Let $\chi(G)$ be the usual chromatic number of the graph G. For any graph G, $\chi la(G) \geq \chi(G)$. In this talk we discuss some results on $\chi la(G)$.

On zero-divisor graphs of local principal ideal rings

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Let R be a commutative ring with identity. The *zero-divisor graph* of R, denoted by $\Gamma(R)$, is the simple graph whose vertices are the nonzero zero-divisors of R and two distinct vertices x, y are adjacent if and only if $xy = 0$. In this talk, for a finite local principal ideal ring R, we discuss the structure of $\Gamma(R)$, determine the vertex connectivity of $\Gamma(R)$, characterize the vertices of minimum degree and the minimum cut-sets of $\Gamma(R)$.

Trees with the reciprocal eigenvalue property

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It was shown in 2006 that among the nonsingular trees T (whose adjacency matrix $A(T)$ is nonsingular), the corona trees (trees that are obtained by taking any tree T and then adding a new pendant vertex at each vertex of T) are the only ones which satisfy the reciprocal eigenvalue property (λ is an eigenvalue of $A(T)$ if and only if $\frac{1}{\lambda}$ is an eigenvalue of $A(T)$). A general question remained open. Can there be a tree which has at least one zero eigenvalue and whose nonzero eigenvalues satisfy the reciprocal eigenvalue property? We prove that there are no such trees with at least two vertices. The proof is a beautiful application of the product of graphs. This talk is based on joint work with Debabrota Mondal and Sukanta Pati.

Fractals with Applications in Graph Theory

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Recently, many researchers are visualized through fractals and graph theory. In several scientific disciplines, fractals and graph theory are applicable and resolve complex problems from different areas of science and engineering. Due to this reason, fractals and graph theory are most fashionable topics of exploration by mathematicians, scientists and engineers. The main focus in this talk is to provide a glimpse of fractals with applications in graph theory.

On some Topological descriptors of Zero-divisor graphs

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The method of encoding information using topological molecular descriptors and resolvability parameters on the molecular structure has a low computing cost and strong predictive potential. Additionally, these molecular descriptors also give information for easy identification of structural properties. Indeed, these are algebraic quantities connected with the chemical structure that correlates it with various physical characteristics. Zero-divisor graph is a geometric representation of a commutative ring. Zero-divisor graph of a ring R is denoted by $\Gamma(R)$, defined by a graph whose vertices are all elements of zero-divisor set of a ring R, and two distinct vertices z_1 and z_2 are adjacent if and only if $z_1.z_2 = 0$. This article examines the edge metric dimension and topological characteristics of $\Gamma(R)$. In general, we discuss $edim(\Gamma(\mathbb{Z}_m))$, where $\mathbb{Z}_m$ is a ring of integers modulo m and $edim(\Gamma(\mathbb{Z}_m[i]))$, where $\mathbb{Z}_m[i]$ is a ring of Gaussian integers modulo m. Moreover, we also study the Zagreb and Sombor index of $\Gamma(\mathbb{Z}_m[i])$.

On Enhanced Power Graph of Finite groups

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The enhanced power graph of a finite group G, denoted by $\mathcal{P}_E(G)$, is a simple undirected graph whose vertex set is G and two distinct vertices x, y are adjacent if $x, y \in \langle z \rangle$ for some $z \in G$. In this talk, we shall see interconnections between graph theoretic properties of $\mathcal{P}_E(G)$ and the algebraic properties of the underlying group G. Several results on the enhanced power graph of finite groups will be discussed

Graph Directed Coalescence Hidden Variable Fractal Interpolation Functions

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In this talk we discuss the constructions of fractal interpolation functions (which is self-affine as well non-self-affine in nature) realizing a given directed graph.

Coloring (P_5, C_4) -free graphs with $\Delta - 1$ colors

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Brooks' theorem states that for a graph G with $\Delta(G) \geq 3$, then $\chi(G) \leq \max\{\Delta(G), \omega(G)\}$. Borodin and Kostochka conjectured a result strengthening Brooks' theorem, stated as, if $\Delta(G) \geq 9$, then $\chi(G) \leq \max\{\Delta(G) - 1, \omega(G)\}$. This conjecture is still open for general graphs. In this talk, we present that the conjecture is true for graphs having no induced path on five vertices and no induced cycle on four vertices. In other words, we say that (P_5, C_4)-free graphs are $\Delta - 1$-colorable.

Cryptographic Application of Graph Theory

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have begun in 1736 with the publication of Euler's solution to the Konigsberg bridge problem. Euler (1707-1782) is known as the father of graph theory. A graph is an ordered pair G = (V, E) comprising a set V of vertices or nodes together with a set E of edges or lines joining the vertices of V. Graph theory is applied in many diverse areas as social sciences, linguistics, physical sciences, computer science, communication engineering, and others. Because of this diversity of applications, it is useful to develop and study the subject in abstract terms and to interpret its results in terms of the objects of any particular system in which one may be interested, for example, in Cryptography. Cryptography is the art and science of secret writing, which deals with means and methods of data conversation in a secure form so that unauthorized persons cannot use the data, the authenticity of data can be established, modification of data can be avoided, and the generator cannot repudiate the data. Thus, the main goals of cryptography are confidentiality, authenticity, data integrity, and non-repudiation. The history of cryptography is quite old and interesting. The idea of cryptography appeared long ago in the form of message communication through symbols, puzzles, etc. The word cryptography is made up of two components: "Kryptos", which means hidden, and "graphic"; which means writing. In ancient times, cryptography was used to safeguard military and diplomatic communications. For example, the famous Roman emperor Julius Caesar used shift cipher to deliver messages to his generals securely; he didn't trust his messengers. So, he replaced every character with a third character using a circular right shift, i.e., replace A with D, B with E, and so on through the alphabet. To get the original message, one needs to apply the third circular left shift operation. The original message is called quot;plaintextquot;. The message in hidden form is called "ciphertext";. Encryption is any procedure to convert plaintext into ciphertext. Decryption is any procedure to convert ciphertext into plaintext. A Key is a piece of information used for encryption and Decryption. The key used for encryption is the encryption key, and the key used for Decryption is the decryption key. Ciphertext space, Plaintext space, Keyspace, Encryption function, and Decryption function all together form a cryptosystem. There are two types of cryptosystems. The first is a private key (symmetric key) cryptosystem, and the second is a public key (asymmetric key) cryptosystem. If the encryption and decryption keys are both the same, then it is known as a private key cryptosystem or a public key cryptosystem. A public key cryptosystem is based on some well-known mathematical hard problem. A hard problem is a problem which is computationally infeasible to solve in polynomial time. Many public key cryptosystems exist in the literature based on different mathematical problems, e.g., Knapsack, RSA, ElGamal, XTR, Rabin, ECC, etc. Cryptosystems can be designed using the concept of graph theory also for example, perfect code in the graph, complete graph, Hamiltonian path in a graph, dominating set in a graph, etc., can be used to design cryptosystems for secure communication.

Discrete version of Guass-Bonnet Theorem

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Discrete differential geometry is the study of discrete counterparts of notions in differential geometry. In this talk we discuss the celebrated Gauss-Bonnet theorem in discrete setup. The above theorem is very interesting because it relates a differential geometric quantity, the boundary curvature, with a topological invariant, the Euler characteristic. In differential geometry, curvature needs a differentiable structure, while Euler characteristic does not. I will define all basic notions needed to state the theorem in discrete setup and at the end mention a few more interesting results and references in the area of discrete differential geometry.

Sampling of signals in shift-invariant spaces and graph signals

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Sampling problem plays a significant role in signal and digital data processing. The aim of this talk is to find a sampling set $X = \{x_k : k \in I\}$, where I is an index set, such that every f in a given class of signals V can be reconstructed uniquely and stably from its samples $\{f(x_k) : x_k \in X\}$. In practical applications, measurement apparatus only produces the average samples. Thus, it is natural to ask whether and how f in V can be reconstructed uniquely and stably from its average samples. The sampling problem in shift-invariant space $V(\phi)$ has been extensively studied in literature. In this talk, we discuss the sampling sets in $V(\phi)$ and provide iterative reconstruction algorithm for reconstruction of a signal f in $V(\phi)$

from its average samples. Further, we also discuss the sampling problem for bandlimited graph signals.

Uniformly Convergent Computational Method for Singularly Perturbed Time-Dependent Semilinear Convection-Diffusion Equations with Discontinuous Data

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This work focuses on a class of singularly perturbed semilinear parabolic partial differential equations (PDEs) of convection-diffusion type with discon- tinuous source function. PDEs of such types often appear in the mathematical modeling of various phenomena in physics and engineering, for instance, semi- conductor device modeling. As the analytical solutions of these PDEs often possess a weak interior layer and a boundary layer, finding uniformly conver- gent numerical solutions becomes a challenging task. To fulfill this objective, we devise a higher-order time-accurate computational method for solving the considered nonlinear PDE on a suitable layer-adapted mesh. Firstly, we discuss the existence, stability, and asymptotic behavior of the analytical solution of the continuous nonlinear problem. We then show that the discrete solution is stable and -uniformly convergent in the supremum norm. This finding is confirmed by the numerical experiment.

Some complexity results for Injective Coloring Problem and Injective Edge Coloring Problem

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Graph coloring is one of the most well-studied and active area of research in graph theory. In this talk, we will explore some complexity results of two interesting graph coloring problems, namely Injective Coloring Problem and Injective Edge Coloring Problem. Given a graph G and a positive integer k, the Injective Coloring Problem is to decide whether G admits an injective k-coloring, i.e. a vertex coloring using k colors such that no two vertices having a common neighbor receive the same color. Whereas, the Injective Edge Coloring Problem is to decide whether G admits an injective k-edge-coloring, i.e. an edge coloring using k colors such that any two edges e and f receive distinct colors if there exists an edge $g = x\,y$ different from e and f such that e is incident on x and f is incident on y. For both problems, we will discuss some interesting linear time algorithms for the subclasses of bipartite graphs.

Some skew differential identities on Prime rings

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Consider a prime ring R. An additive mapping d on R is said to be skew derivation associated with an automorphism α on R if $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in R$. For example, the mapping $d : R \to R$ defined as $d(x) = ax - \alpha(x)a$ for all $x \in R$ and for given $a \in R$, is a skew derivation on R. A polynomial equation on R containing skew derivation and automorphism is said to be skew differential identity. Its form look like $p(x_i, d(x_i), \alpha(x_i)) = 0$ where x_i are indeterminates. In this talk we consider some identities containing skew derivation d and discuss the structure of d as well as the structure of R.

$PGL(2, q)$-orbits of lines of $PG(3, q)$ and binary quartic forms

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et $PG(3, q)$ denote the 3-dimensional projective space defined over a finite field $\mathbb{F}_q$ of order q. Let C be the twisted cubic in $PG(3, q)$ defined by the image of the embedding

$$PG(1, q) \hookrightarrow PG(3, q), \quad (s, t) \mapsto (s^3, s^2t, st^2, t^3).$$

The twisted cubic is left invariant under the action of a group $G \leq PGL(4, q)$, which is isomorphic to $PGL(2, q)$. The standard action of $PGL(2, q)$ on the points (s, t) of $PG(1, q)$, induces an action on the points of C, and this action extends to an action of $PGL(2, q)$ on $PG(3, q)$. A binary quartic form over $\mathbb{F}_q$ is a homogeneous polynomial of degree 4 in variables X, Y, whose coefficients are in $\mathbb{F}_q$. The group $PGL(2, q)$ acts on binary quartic forms by $(g \cdot f)(X, Y) = f(dX - bY, aY - cX)$, where $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ad - bc \neq 0$.

In this talk, we shall first discuss about the $PGL(2, q)$-orbits classification of the binary quartic forms over $\mathbb{F}_q$, when the char$(\mathbb{F}_q) \neq 2, 3$. Then we shall discuss how to use these results to obtain the $PGL(2, q)$-orbits classification of the lines of $PG(3, q)$. This is a joint work with Dr. Krishna Kaipa and Dr. Nupur Patanker.

Characterizing Parabolic Hyperplanes of the Hyperbolic and Elliptic Quadrics in $PG(2n + 1, q)$

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In this talk, we discuss a combinatorial characterization of the family of parabolic hyperplanes of a hyperbolic (respectively, elliptic) quadric of $PG(2n+1,q)$, using their intersection properties with the points and subspaces of codimension 2.

On Connectivity Status of Fuzzy Graphs

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Weighted networks are part of our daily life. Topology of networks are best viewed by graphs. Reliable performance and stability of networks are well related with their connectivity. Connectivity parameters are indicators of the connectivity of networks. This talk aims to discuss a connectivity parameter named connectivity status related to fuzzy graphs. Connectivity analysis using connectivity status will be discussed. Connectivity status applied to a problem of bandwidth allocation will also be presented.

Finitely Generated Minimal Linear Code

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We find one type of finitely generated minimal linear code over the ring Z_{2n}. We show that if the components of the vectors belongs to the ideal $< n >$ then with a given condition the module generated by this vectors must be minimal linear code.

Isolated Gaps in Numerical Semigroups

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A subset S of the set $\mathbb{N}$ of nonnegative integers is a *numerical semigroup* if S is closed under addition, S contains 0, and the complement of S in $\mathbb{N}$ is finite. A nonempty subset A of a numerical semigroup S is a *set of generators* of S if $S = \{k_1a_1 + \cdots + k_na_n : n, k_1, \ldots, k_n \in \mathbb{N}$ and $a_1, \ldots, a_n \in A\}$. The *embedding dimension* of a numerical semigroup is the cardinality of its minimal set of generators. Given a numerical semigroup S, an element $s \in \mathbb{N} \setminus S$ is an *isolated gap* in S if the set $\{s-1, s+1\}$ is a subset of S. A numerical semigroup is *perfect* if it does not contain any isolated gaps. We discuss some basic properties of isolated gaps in a numerical semigroup. Using these, we determine the cardinality of the set of isolated gaps in a numerical semigroup of embedding dimension two. We further give a formula to calculate the smallest isolated gap in a numerical semigroup of embedding dimension two. We finally discuss a method to generate some examples of perfect numerical semigroups of embedding dimension three.

Impulse Minimization, Index Reduction, and Column Regularization for Descriptor Systems

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Index of a descriptor system reflects the intensity of impulses in it. The necessary and sufficient condition on descriptor systems having index at most a specific value and column regularity are established. Additionally, a necessary condition is proposed for the existence of an output feedback to reduce the index of the system to a pre-assigned desired value and column-regularize. Furthermore, the necessary and sufficient condition is derived for the existence of an output feedback to reduce the system's index at most two. All theoretical conditions are proposed directly in terms of system matrices. Orthogonal transformations are used for design procedure and proofs. Numerical examples are presented to support the outcome of this paper.

How to Make Knockout Tournaments More Popular?

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Given a mapping from a set of players to the leaves of a complete binary tree (called a seeding), a knockout tournament is conducted as follows: every round, every two players with a common parent compete against each other, and the winner is promoted to the common parent; then, the leaves are deleted. When only one player remains, it is declared the winner. This is a popular competition format in sports, elections, and decision-making. Over the past decade, it has been studied intensively from both theoretical and practical points of view. Most frequently, the objective is to seed the tournament in a way that "assists" (or even guarantees) some particular player to win the competition.

I will talk about a new objective, which is very sensible from the perspective of the directors of the competition: maximize the profit or popularity of the tournament. Specifically, we associate a "score" with every possible match, and aim to seed the tournament to maximize the sum of the scores of the matches that take place. I will focus on the case where we assume a total order on the players' strengths, and will briefly discuss a wide spectrum of results on the computational complexity of the problem, along with some open problems.

Algebraic invariant of edge ideals of some graphs

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We are interested in the complete classification of Stanley-Reisner rings of path (cycle) join graphs with 2-linear resolution along with their Hilbert series and Betti numbers. Also, we present combinatorial formulae for the regularity (Castelnuovo- Mumford), projective dimension of some path join graphs. Homological invariants of

some other graphs structures are discussed along with their applications to graphs rinsing from algebraic structues

Image Denoising using Artificial Neural Network's

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Image enhancement aims in improving the quality of a digital image needed for visual inspection or for automated image processing operations. Noises generated from various sources corrupt images, and their visual quality goes down significantly. In real systems it is not possible to remove noise completely; therefore noise suppressing methods are developed. These methods are based on the designing of filters which suppress the noise along-with some signal components. Most of these methods are iterative in nature; therefore they take time before output is available. To counteract such problem Artificial Neural Network (ANN) can be used. This paper addresses ANN based noise removal method and simulation results are presented showing noisy and de-noised images. The performance of the de-noised image is measured in terms of PSNR and SSIM.

A Note on a Conjecture of Gao and Zhuang for Groups of Order 27

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Let G be a finite group written multiplicatively with the identity element 1. Let $F(G)$ denote a free abelian monoid with basis G (as a set), whose elements are called sequences over the group G. In 1977, Olson and White studied extensions of the widely known Davenport constant for the first time in the case of non-abelian groups. The small Davenport constant $d(G)$ is the maximum positive integer l such that there exists a product-one free sequence over G of length l. One can observe that $d(G) + 1 \leq |G|$. The Gao constant $E(G)$ is the minimum positive integer k such that every sequence S over G of length k has a non-empty

product-one subsequence T of length $|G|$. In 2005, Gao and Zhuang conjectured that for any finite non- abelian group G, we have $E(G) = d(G) + |G|$. For an odd prime p, let H_{p^3} denote the Heisenberg group of order p^3 with the exponent p. In this talk, we will discuss our recent results based on this conjecture. Along with this, we will see some combinatorial properties of H_{p^3} using its z-classes.